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A Hyperboloidal Constricted Tube Model of Porous Media

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In recent years, numerous publications on flow of fluids through porous media have emphasized the need for geometric models to describe and analyze transport processes in the pores of the media. A few models, both old and new, are indeed available for this purpose, although the particular choice usually depends on the specific application under consideration. The highly simplified capillaric models serve only a limited purpose, as the curvature of the actual pores may be critical in most applications. The conceptually simple, but quite versatile, cell models (such as the sphere-in-cell model of Happel) have been used extensively in the study of heat and mass transfer in packed beds and lately have been shown to be highly successful in predicting deposition (of particles from suspensions) in porous structures (Rajagopalan and Tien 1976, 1979), but they do not account for the converging-diverging nature of the pores.

In 1973, Payatakes, Tien, and Turian proposed a periodically constricted tube model (PCT) based on Petersen's (1958) preliminary suggestion to account for the effect of neighboring grains and the converging-diverging nature of the flow channels. This PCT model has been used for studying deposition of particles in deep bed filtration (for instance, Payatakes, Tien, and Turian 1973, 1974, and Rajagopalan and Tien 1979). Another PCT model, with sinusoidal walls, has been introduced since then by Fedkiw and Newman (1977) for studying mass transfer at high Peclet numbers, and other variations have also been examined (Azzam and Dullien 1977).

These models are potentially quite useful, but their use is severely restricted by the difficulty in obtaining a *closed-form* solution to the corresponding hydrodynamic problem. The solution of the Navier-Stokes equation in these cases is difficult and requires numerical methods even for the creeping flow case. Certain applications, such as trajectory calculations in deposition studies, require the velocity fields for statistically dissimilar tubes. This implies that the above problem has to be solved (numerically) for numerous constricted tubes. Although a collocation solution to this problem, for one particular choice of tube geometry, is now available (Neira and Payatakes 1978), collocation expansion coefficients are still needed for arbitrary geometries of the constricted tubes. The above considerations make it clear that a simple, but effective, model with a closed-form expression for the stream function is highly desirable.

The purpose of this note it to present such a model. The model proposed here retains the converging-diverging character of the pore, but does not consider the pores (and hence the flow through them) to be periodic, since in many applications periodicity is not required. Geometric models of porous media have been used frequently to describe and predict the average macroscopic response of the media to the processes that occur within them (including, for example, the pressure drop, mass or heat transfer rates, and residence times). We found that the new model predicts the macroscopic parameters of the flow with the same degree of accuracy as obtained through PCT models. Besides, one is not restricted to a choice of geometrically similar constricted tubes in the case of the present model (in contrast to the PCT models). The new model has three arbitrary geometric parameters, as the previous ones, but one may consider a collection of constricted tubes of any combination of arbitrary geometric parameters, and the flow field and the pressure drop in all of them can be readily obtained as functions of these parameters.

MODEL FORMULATION

We shall consider a packed bed to be composed of statistically identical unit bed elements (UBE) each of which in turn consists of an array of constricted tubes, as originally proposed by Payatakes et al. (1973). The walls of these tubes are generated by a one-sheeted hyperboloid of revolution about the axis of symmetry. Each tube, hereafter called Hyperboloidal Constricted Tube (HCT), is characterized by the following three geometric parameters: the length, l, the throat diameter, d_t, and the entrance (or exit) diameter, d_e. These parameters can be obtained by the same technique used to calculate the PCT model parameters.

A sketch of a typical hyperboloidal constricted tube is presented in Figure 1. The appropriate coordinate system in this case is the set of oblate spheroidal coordinates (ξ, η, ϕ) with the origin at the center of the tube.[1] Figure 1 also shows the corresponding circular cylindrical coordinates (r, z, ϕ). The relation between the oblate spheroidal coordinates and the cylindrical coordinates and the correspondence between the HCT dimensions and the oblate spheroidal coordinate boundaries are conveniently summarized in Table 1.

Since low Reynolds number flow through such constricted tubes is of primary interest in many applications, consider creeping motion of a viscous fluid through an HCT in the negative z-direction. In this case, the equation of motion reduces to

$$E^4\psi = 0 \tag{1}$$

where ψ is the stream function and the operator E^2 is defined by

$$E^2 = \frac{1}{c^2(\lambda^2+\zeta^2)}\left[(\lambda^2+1)\frac{\partial^2}{\partial\lambda^2} + (1-\zeta^2)\frac{\partial^2}{\partial\zeta^2}\right] \tag{2}$$

with $\lambda = \sinh\,\xi$ and $\zeta = \cos\,\eta$.

The stream surfaces coincide with the surfaces of the confocal hyperboloids, η = constant; thus $\psi = \psi(\zeta)$ only.

Equation (1) is to be solved subject to the following boundary conditions

$$\psi = 0 \text{ at } \zeta = 1 \text{ (by definition)} \tag{3}$$

$$\psi = q/2\pi \text{ at } \zeta = \zeta_0;\ \zeta_0 \geq 0 \tag{4}$$

where q is the volumetric flow rate through the HCT, and

$$\frac{d\psi}{d\zeta} = 0 \text{ at } \zeta = \zeta_0;\ \zeta_0 \geq 0 \tag{5}$$

The above three conditions, which are for the upper half of the tube ($0 \leq \eta \leq \pi/2$), have to be modified slightly for the lower half

$$\psi = 0 \text{ at } \zeta = -1 \tag{3'}$$

$$\psi = q/2\pi \text{ at } \zeta = \zeta_0;\ \zeta_0 < 0 \tag{4'}$$

$$\frac{d\psi}{d\zeta} = 0 \text{ at } \zeta = \zeta_0;\ \zeta_0 < 0 \tag{5'}$$

This problem has a unique solution, which is given in Table 2 (also see Happel and Brenner 1965).

On account of symmetry, the pressure drop experienced by the fluid flowing through the HCT is

$$-\Delta p = 2(p_e - p_o) \tag{6}$$

where p_e is the *average* pressure at the entrance of the tube and p_o is the *uniform* pressure at the throat of the tube, $\lambda = 0$. The pressure drop, $-\Delta p$, can be obtained from the pressure gradients, which in turn are related to the stream function by

$$\frac{\partial p}{\partial\lambda} = \frac{\mu}{c(\lambda^2+1)}\frac{\partial}{\partial\zeta}(E^2\psi) \tag{7}$$

and

$$\frac{\partial p}{\partial\zeta} = \frac{\mu}{c(1-\zeta^2)}\frac{\partial}{\partial\lambda}(E^2\psi) \tag{8}$$

Velocity components in oblate spheroidal coordinates are

$$v_\xi = \frac{h_\eta}{c(1+\lambda^2)^{1/2}}\frac{\partial\psi}{\partial\zeta} \tag{9}$$

and

$$v_\eta = \frac{h_\xi}{c(1-\zeta^2)^{1/2}}\frac{\partial\psi}{\partial\lambda} \tag{10}$$

where $h_\eta = h_\xi = 1/[c(\lambda^2+\zeta^2)^{1/2}]$

The resulting expressions for pressure drop, Δp, and velocity, v, are listed in Table 2. It is convenient to present the pressure drop in terms of a friction factor, f_t, which we shall define as

$$f_t = -(\Delta p/l)[d_t/2\rho v_t^2] \tag{11}$$

where v_t is the average velocity at the throat. Now let the throat

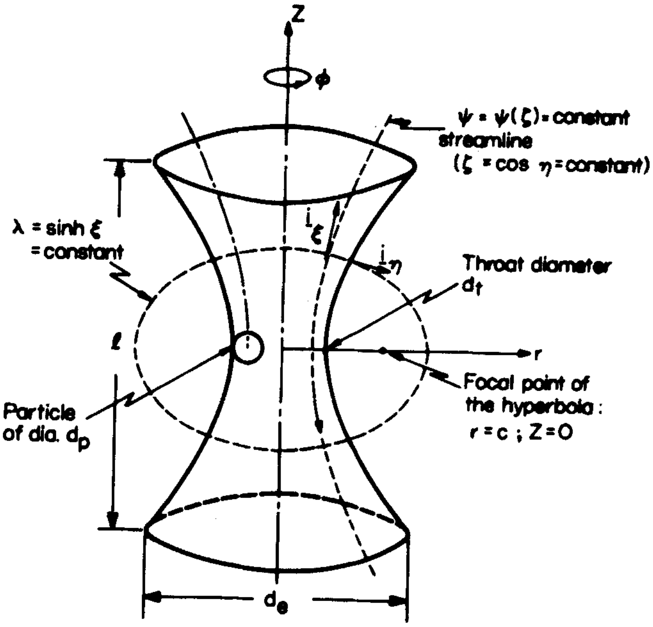

Figure 1. Geometrical parameters of the hyperboloidal constricted tube and the oblate spheroidal coordinate system.

Reynolds number be

$$N_{Re,t} = d_t v_t/\nu \tag{12}$$

Then it can be shown that

$$f_t \cdot N_{Re,t} = 6\,g(\lambda_0, \zeta_0) \tag{13}$$

where the function g is given in Table 2. Equation (13) gives the pressure drop as a function of the geometrical parameters of the model.

COMPARISON WITH THE PCT MODEL

Before presenting an application of this model, we shall briefly compare the pressure drop across an HCT, and the

TABLE 1. COORDINATE TRANSFORMATIONS AND MODEL PARAMETERS

A. RELATION BETWEEN OBLATE and SPHEROIDAL COORDINATES AND CYLINDRICAL COORDINATES

Cylindrical Coordinates (r, z, ϕ)		Oblate Spheroidal Coordinates (ξ, η, ϕ)	
z	=	$c\sinh\xi\cos\eta$:	$0 \leq \xi < \infty$
r	=	$c\cosh\xi\sin\eta$:	$0 \leq \eta \leq \pi$
ϕ	=	ϕ	: $0 \leq \phi \leq 2\pi$; $c > 0$

The metric coefficients:

$h_\xi = h_\eta = 1/[c(\cosh^2\xi - \sin^2\eta)^{1/2}]$

$h_\phi = 1/(c\cosh\xi\sin\eta)$

Typical unit vectors are depicted in Fig. 1. The unit vector i_ϕ is directed into the page.

B. RELATION BETWEEN THE HCT DIMENSIONS AND THE CORRESPONDING COORDINATE BOUNDARIES

$\xi_0 = \cosh^{-1}(d_e/d_t)$	;	$\lambda_0 = \sinh\xi_0$
$\eta_0 = \tan^{-1}[(d_t/l)\sinh\xi_0]$	;	$\zeta_0 = \cos\eta_0$
$c = d_t/(2\sin\eta_0)$		

OR

$\lambda_0 = [(d_e/d_t)^2 - 1]^{1/2}$	;	$\xi_0 = \sinh^{-1}\lambda_0$
$\zeta_0 = [(d_t\lambda_0/l)^2 + 1]^{-1/2}$	;	$\eta_0 = \cos^{-1}\zeta_0$
$c = l/2\lambda_0\zeta_0$		

[1] The choice of this coordinate system is suggested by the chosen geometry of the constricted tube. In fact, Petersen (1958) employed the same coordinate system in his work, although to solve the steady state, stagnant diffusion problem in porous media. The analytical solution for this problem is straightforward since the resulting equation is separable (see also Moon and Spencer 1971).

TABLE 2. EXPRESSIONS FOR STREAM FUNCTION, VELOCITY FIELD, PRESSURE DROP AND FRICTION FACTOR

Stream Function

$$\psi(\zeta) = \frac{q}{2\pi}\frac{\{(1-3\zeta_0^2) - \zeta(\zeta^2-3\zeta_0^2)\}}{[(1+2\zeta_0)(1-\zeta_0)^2]} \quad 0\le z\le\frac{l}{2};\ \zeta_0\ge 0$$

$$= \frac{q}{2\pi}\frac{\{(1-3\zeta_0^2) + \zeta(\zeta^2-3\zeta_0^2)\}}{[(1-2\zeta_0)(1+\zeta_0)^2]} \quad \frac{-l}{2}\le z\le 0;\ \zeta_0<0$$

Velocities

$$v_\eta = \frac{h_\xi}{c(1-\zeta^2)^{1/2}}\frac{\partial\psi}{\partial\lambda} \equiv 0$$

$$v_\xi = \frac{h_\eta}{c(1+\lambda^2)^{1/2}}\frac{\partial\psi}{\partial\zeta}$$

$$= \frac{\pm 3D\,(\zeta^2-\zeta_0^2)}{c^2(\lambda^2+\zeta^2)^{1/2}(1+\lambda^2)^{1/2}};\ \begin{cases} + \text{ if } \zeta<0;\ \text{lower half} \\ - \text{ if } \zeta\ge 0;\ \text{upper half}\end{cases}$$

$$D = \frac{q}{2\pi(1+2\zeta_0)(1-\zeta_0)^2};\ \zeta\ge 0$$

$$= \frac{q}{2\pi(1-2\zeta_0)(1+\zeta_0)^2};\ \zeta< 0$$

Pressure Drop and Friction Factor

$$-\Delta P = \frac{12D\mu}{c^3}\left[\frac{\lambda_0}{1+\lambda_0^2} + \tan^{-1}\lambda_0\right];\ D \text{ for } \zeta\ge 0$$

$$f_t = 6g(\lambda_0,\zeta_0)/N_{Re,t}$$

$$g = \frac{(1+\zeta_0)^2}{(1+2\zeta_0)}\frac{1}{\lambda_0\zeta_0}\left[\frac{\lambda_0}{1+\lambda_0^2} + \tan^{-1}\lambda_0\right];\ \zeta_0\ge 0$$

Note: The expressions for $\psi(\zeta)$ and D for the upper half of the HCT ($\zeta\ge 0$) can be used for the lower half also if one takes ζ and ζ_0 to be always positive.

velocity fields in it, with the corresponding values obtained with the PCT model. Consider flow through a tube with the following geometry:

$$d_t^* = d_t/l = 0.364$$
$$d_c^* = d_c/l = 0.860 \qquad (14)$$
$$l = 0.0711 \text{ cm}$$

These dimensions, which correspond to a "typical pore" in a randomly packed bed of sand grains (with average grain diameter of 0.0714 cm), were used by Payatakes, Tien, and Turian (1973) in their calculations. The friction factor, f_s, defined by Payatakes et al. is based on the superficial velocity, v_s, of the fluid in the bed and is given by

$$f_s = -(\Delta p/l)[\langle d_g\rangle/2\rho v_s^2] \qquad (15)$$

where $\langle d_g\rangle$ is the average grain diameter. Such a definition, which is not specified directly in terms of the individual constricted tubes, is sufficient in their case since all the constricted tubes in a Unit Bed Element become identical when transformed to dimensionless geometric parameters. As we stated earlier, the HCT model of this paper is free of such restrictions, and it is more appropriate to define the friction factor for each tube as in Equation (11). The characteristic velocity and diameter in such a definition are arbitrary and we have chosen the values at the throat of the tube for our definition (Equation 11). When the packed bed is viewed as a collection of such tubes, the overall friction factor (or, correspondingly, the pressure drop) can be computed by appropriately averaging Equation (11).

But to compare our model with that of Payatakes et al., the correspondence between f_t and f_s must be established. That is, we must reformulate their friction factor, f_s, in terms of the throat velocity and diameter of their PCT. This can be done quite easily and one obtains

$$f_t\cdot N_{Re,t} = f_s N_{Re,s}[\pi N_c\, d_c^3 d_t/4\langle d_g\rangle^2] \qquad (16)$$

where the superficial Reynolds number, $N_{Re,s}$, is $\langle d_g\rangle v_s/\nu$ and N_c is the number of constrictions per unit cross sectional area of the bed. Payatakes, Tien, and Turian (1973) obtained the following equation for f_s when $N_{Re,s} \lesssim 5$

$$f_s\cdot N_{Re,s} = 460 \qquad (17)$$

For the geometry specified in Equation (14), with $N_c = 171$ cm^{-2} and $\langle d_c^3\rangle = 21.429\times 10^{-6}$ cm^3, Equations (16) and (17) lead to the following expression for the friction factor, f_t

$$f_t\cdot N_{Re,t} = 6.7208 \qquad (18)$$

Now, the predictions of the new model for the above parameters can be determined easily from the information provided in Tables 1 and 2. Firstly, the oblate spheroidal boundaries that correspond to the dimensions in Equation (14) follow immediately from the transformations in Table 1

$$\lambda_0 = 2.1406$$
$$\zeta_0 = 0.78882 \qquad (19)$$
$$c = 210.5\ \mu\text{m} = 0.02105 \text{ cm}$$

The above quantities define the geometry of the required HCT model. The corresponding friction factor can be computed from the equations in Table 2 and is given by

$$f_t\cdot N_{Re,t} = 6.6928 \qquad (20)$$

in excellent agreement with Equation (18).

It is also instructive to compare the velocity profiles in a PCT and the corresponding HCT. As an example, consider the velocity components in the HCT corresponding to the tube dimensions treated in Neira and Payatakes (1978). For the purpose of comparison, the non-vanishing component v_ξ is resolved into axial (v_z) and radial (v_r) components and both v_z and v_r are scaled with respect to v_t, the average velocity at the throat. The results are shown in Figures 2a and b along with the velocity profiles obtained by Neira and Payatakes for their PCT model. A discussion of these will be presented later in this paper.

AN APPLICATION OF THE MODEL

We shall present briefly an important application of the above model as an illustration of its use. One of the immediate uses we foresee is in studying particle deposition and the subsequent alteration of the structure of the flow passages in packed beds, a problem of growing interest.

Consider a liquid suspension flowing through a filter bed. The deposition of particles onto the surface of the grains of the filter

TABLE 3. INTERCEPTION EFFICIENCY USING HCT MODEL, COMPARED WITH PCT MODEL

$d_t^* = 0.3;\ d_c^* = 0.8$

$\eta_I \times 10^2$

	d_p = 2.75 μm		d_p = 4.5 μm		d_p = 9.0 μm	
l, cm	PCT*	This Work	PCT*	This Work	PCT*	This Work
0.0436	0.1900	0.2001	0.5005	0.5262	1.9181	2.0087
0.0520	0.1342	0.1414	0.3543	0.3728	1.3673	1.4336
0.0624	0.0935	0.0986	0.2475	0.2606	0.9609	1.0087
0.0737	0.0672	0.0709	0.1782	0.1877	0.6952	0.7304

*Based on the collocation solution of Neira and Payatakes (1978).

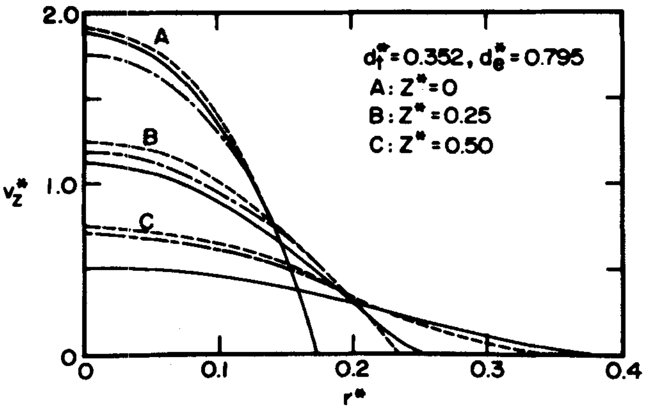

Figure 2a. Axial velocity profiles in a constricted tube: ———— This work (HCT); ----- Neira and Payatakes, 1978 (PCT); -·-·- Payatakes, Tien, and Turian, 1973 (PCT); Throat = A; Entrance or Exit = C.

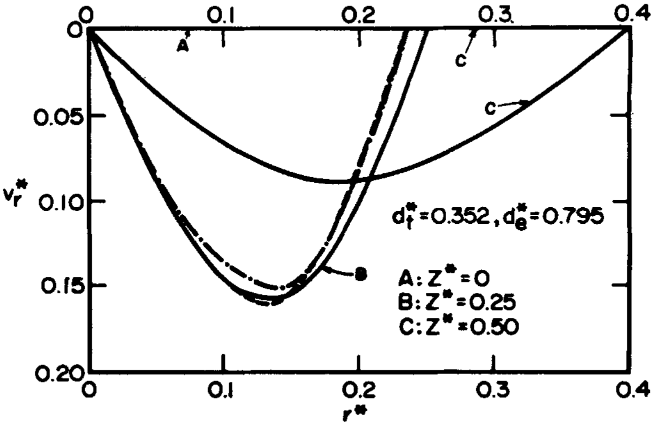

Figure 2b. Radial velocity profiles in a constricted tube: ———— This work (HCT); ----- Neira and Payatakes, 1978 (PCT); -·-·- Payatakes, Tien, and Turian, 1973 (PCT); Throat = A; Entrance or Exit = C.

medium occurs due to various mechanisms, the important ones being diffusion, interception, gravitational collection and collection due to surface forces. Since our primary objective here is to merely provide an example of the application of the model, only the interception efficiency has been computed. An analysis of other mechanisms, which is of interest in separation processes and in particulate fouling, is in progress and will be presented shortly.

Interception efficiency, η_I, is defined as the ratio of the amount of particles collected due to interception to that approaching the collector. By using the limiting trajectory concept (Rajagopalan and Tien 1976), a simple expression for η_I can be obtained.

Let d_p be the average diameter of the suspended particles. If we define N_R as d_p/d_t, then, it can be readily shown that the interception efficiency is given by

$$\eta_I = 1+\{[1-(1-N_R)^2(1-\zeta_0^2)]^{1/2}[1-(1-N_R)^2(1-\zeta_0^2)-3\zeta_0^2] - [1-3\zeta_0^2]\} \div [(1+2\zeta_0)(1-\zeta_0)^2] \quad (21)$$

Thus, η_I depends only on the geometry of the tube and the ratio of the particle diameter to the HCT throat diameter.

The interception efficiencies have been calculated for a few geometrically similar tubes. (Since we have compared our calculations in this paper with the predictions of the PCT model, for which the flow field is available only for similar tubes of a specific geometry, our attention is restricted to the available geometry.) The parameters used are

$$d_t^* = 0.3$$
$$d_e^* = 0.8$$

The values of η_I obtained by using the HCT model are compared with the values obtained by using the collocation solution for the PCT model (of Neira and Payatakes, 1978) and are listed in Table 3.

DISCUSSION

The primary goal of the study of deposition in porous media, the problem that we have chosen as an illustration, is to provide a proper and self-consistent description of the behavior of the packed bed (regarding both the deposition rate and the subsequent pressure drop) as filtration proceeds. Recent studies in this area have focused on the dynamics of the separation process (see Tien et al. 1979, Pendse et al. 1978, and Rajagopalan and Tien 1979). The difficulty here has been the lack of a single geometric (and hydrodynamic) model that could be used to predict both the collection efficiency and the corresponding increase in pressure drop. Rajagopalan and Tien (1976) showed that the sphere-in-cell model is highly effective in doing the former and suggested that the cell model be used in combination with the PCT model, which has been proven quite useful in predicting pressure drops. Some of these are summarized in a recent treatment of the theory of deep bed filtration (Rajagopalan and Tien 1979).

What we seek to examine in this note is the versatility of the HCT model in achieving both with relative ease. Equations (18) and (20) demonstrate that the friction factor for HCT is more than sufficiently close to that of the PCT model, implying that the present model can be used with confidence for predicting pressure drops. The actual velocities of the fluid, however, do differ slightly as shown in Figures 2a and 2b. The differences in the axial velocity, shown in Figure 2a, are not significant, but the radial velocity at the entrance of the hyperboloidal constricted tube is not zero (since the unit vector, $\mathbf{i}_\xi$, is not orthogonal to $\mathbf{i}_r$ except at the throat and along the axis of the HCT). But, the point values of the velocities are not important in most cases, and it is well-known that the mass and heat transfer rates are not very sensitive to minor changes in velocity profiles (see Ruckenstein and Rajagopalan 1980). Indeed, Figure 2b shows that except for short distances near the entrance and exit of the tubes, even the radial velocities are essentially the same in both models.

The deposition rate, represented by the interception efficiency listed in Table 3, is also remarkably close to the values based on the PCT model. In addition, the HCT model offers the convenience of closed-form solutions and arbitrary geometry.

ACKNOWLEDGMENT

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NOTATION

c	=	focal length of the hyperbola, m; (see Figure 1)
d_e	=	entrance diameter of the HCT, m
$\langle d_g \rangle$	=	average grain diameter, m, Equation (15)
d_p	=	average diameter of the suspended particles, m
d_t	=	throat diameter of the HCT, m
f_t	=	friction factor, Equation (11)
h_η, h_ξ	=	metric coefficients (see Table 1)
l	=	length of the HCT, m
N_c	=	number of constrictions per unit cross sectional area of the bed, no./m^2; Equation (16)
N_R	=	aspect ratio, d_p/d_t
$N_{Re,t}$	=	Reynolds number, Equation (12)
q	=	volumetric flow rate through the HCT, m^3/s
r	=	radial coordinate
v_r	=	radial velocity, m/s
v_t	=	average velocity at the throat, $4q/\pi d_t^2$, m/s

v_z = axial velocity, m/s
v_ξ = streamwise velocity, m/s
z = axial coordinate

Greek Letters

ΔP = pressure drop across the HCT, Equation (6), N/m^2
ζ = cos η = transformed coordinate
ζ_0 = HCT coordinate boundary
η = oblate spheroidal coordinate (see Figure 1)
η_I = interception efficiency, Equation (14)
λ = sin$h\xi$ = transformed coordinate
λ_0 = HCT coordinate boundary
μ = viscosity of the medium, kg/m s
ξ = oblate spheroidal coordinate (see Figure 1)
ρ = density of the medium, kg/m^3
ψ = stream function, m^3/s

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Interceptional and Gravitational Deposition of Inertialess Particles On a Single Sphere and In a Granular Bed

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Separation of small dust particles on a single sphere or droplet due to such effects as inertia, interception and gravity, has been investigated by Michael and Norey (1969) Flint and Howarth (1971) Prieve and Ruckenstein (1974) and Nielsen and Hill (1976a, b). Deposition of such particles in a granular bed was treated by Rajagopalan and Tien (1976) and Gutfinger and Tardos (1979). Different flow models for the fluid motion around the sphere such as Stokes, Oseen, and potential flows were used.

Based on the work of Nielsen and Hill (1976a), a general mathematical method is applicable to compute interceptional, gravitational (and certain electrical) deposition efficiencies, if the dust particles are very small or move slow enough so that inertial effects can be neglected. The physical meaning of this assumption is that the small dust particles follow the fluid stream lines exactly. In this note, we present a general mathematical approach for the case of deposition on a single sphere and/or spherical particle (granule) situated in a packed or densely fluidized bed. Also, the meaning of "inertia-less particles" is tested, using a numerical solution for the dust particle trajectory near the surface of the sphere of granules.

As a first step in computing the deposition efficiency, one has to know the stream function characteristic for the fluid flow around a sphere ψ_f, and the stream function of the gravity force field, ψ_G. These functions given by Nielsen and Hill (1976a) are

$$\psi_f = \frac{1}{2} R^2 \sin^2\theta \, h(R, \epsilon) \tag{1}$$

$$\psi_G = \frac{1}{2} R^2 \sin^2\theta \, GaSt \tag{2}$$

Here, $R = r/a$, is the dimensionless radius, $Ga = ag/U_0^2$ is the Galileo number, $St = 2C\rho_p U_0 r^2_p/9\mu a$ is the Stokes number (or